Derivative Graphs

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

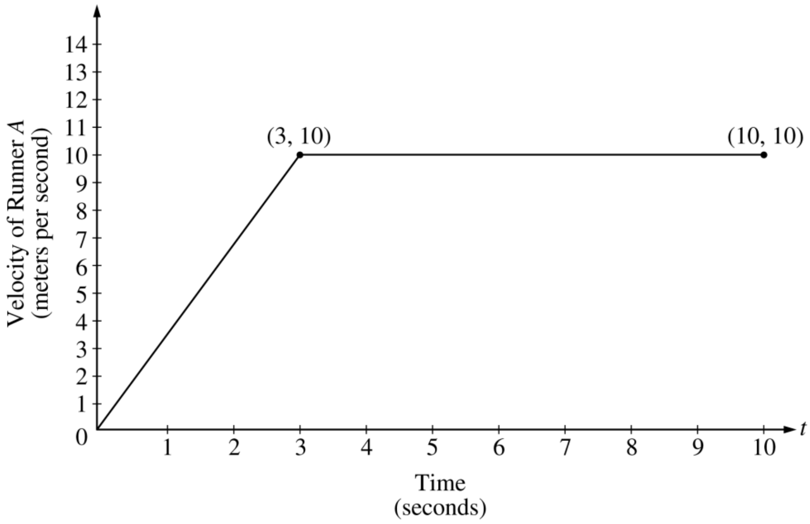
Question 1

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Derivative Graphs, Integration Technique – Geometric Areas

Paper: Part A-Calc / Series: 2000 / Difficulty: Medium / Question Number: 2



2. Two runners, A and B, run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t + 3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

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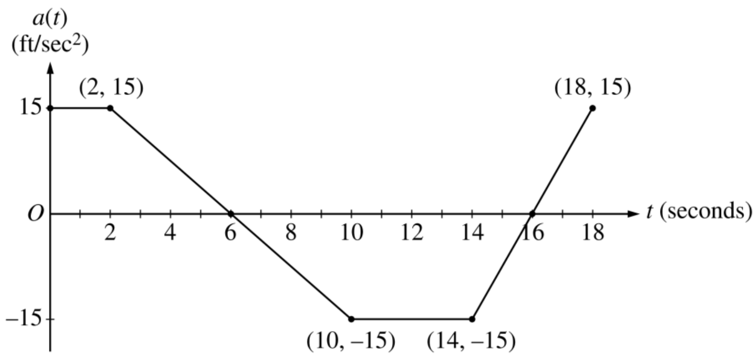
Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Increasing/Decreasing , Integration - Area Under A Curve, Global or Absolute Minima and Maxima, Derivative Graphs

Paper: Part A-Calc / Series: 2001 / Difficulty: Very Hard / Question Number: 3



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

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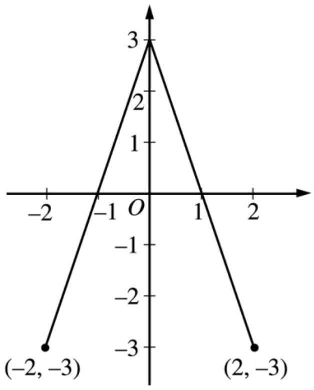
Question 3

Qualification: AP Calculus AB

Areas: Differentiation, Integration, Applications of Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Derivative Graphs, Integration - Area Under A Curve, Increasing/Decreasing , Concavity, Integration Graphs

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 4



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
(Note: The axes are provided in the pink test booklet only.)

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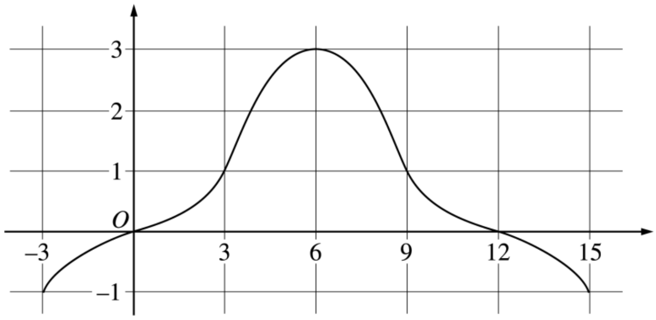
Question 4

Qualification: AP Calculus AB

Areas: Integration, Applications of Integration, Applications of Differentiation, Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Derivative Graphs, Concavity, Increasing/Decreasing , Riemann Sums – Trapezoidal Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Medium / Question Number: 4



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

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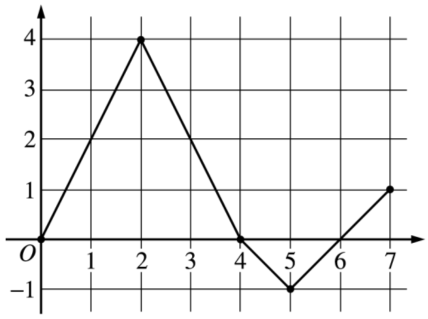
Question 5

Qualification: AP Calculus AB

Areas: Integration, Differentiation, Applications of Differentiation

Subtopics: Derivative Graphs, Rates of Change (Average), Mean Value Theorem, Points Of Inflection, Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (Second), Integration Graphs

Paper: Part B-Non-Calc / Series: 2003-Form-B / Difficulty: Medium / Question Number: 5



Graph of f

5. Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

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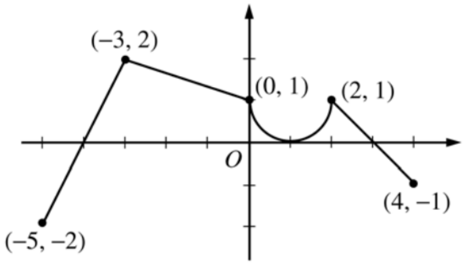
Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Derivative Graphs, Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Points Of Inflection, Integration Graphs

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Somewhat Challenging / Question Number: 5



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Local or Relative Minima and Maxima, Derivative Tables, Derivative Graphs, Fundamental Theorem of Calculus (Second), Points Of Inflection, Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Somewhat Challenging / Question Number: 4

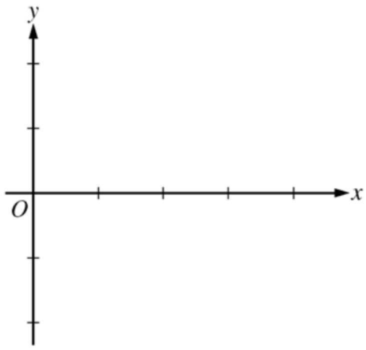
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(Note: Use the axes provided in the pink test booklet.)



(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

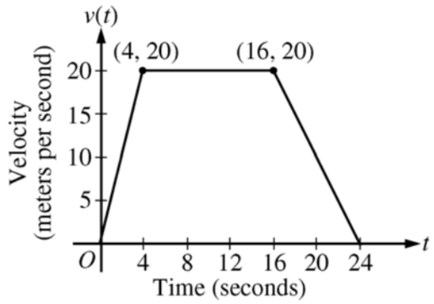
Question 8

Qualification: AP Calculus AB

Areas: Integration, Applications of Integration, Limits and Continuity, Applications of Differentiation

Subtopics: Interpreting Meaning in Applied Contexts, Kinematics (Displacement, Velocity, and Acceleration), Integration Technique – Geometric Areas, Differentiability, Derivative Graphs, Rates of Change (Average), Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Hard / Question Number: 5



5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

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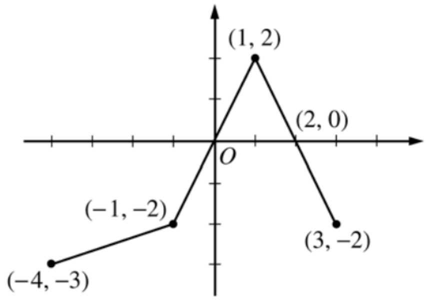
Question 9

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Fundamental Theorem of Calculus (Second), Points Of Inflection, Increasing/Decreasing , Integration Technique – Geometric Areas, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2005-Form-B / Difficulty: Somewhat Challenging / Question Number: 4



Graph of f

4. The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

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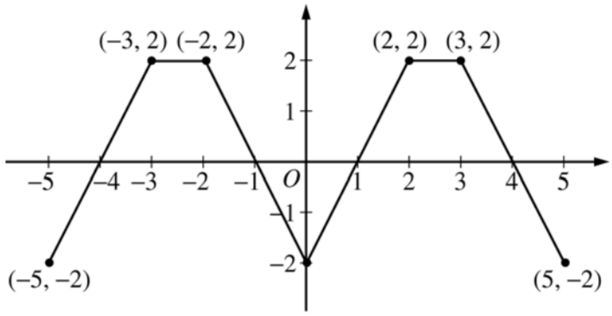
Question 10

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Derivative Graphs, Fundamental Theorem of Calculus (Second), Local or Relative Minima and Maxima, Tangents To Curves

Paper: Part A-Calc / Series: 2006 / Difficulty: Hard / Question Number: 3



Graph of f

3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

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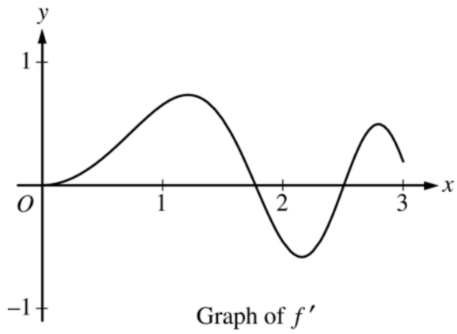
Question 11

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Derivative Graphs, Concavity, Global or Absolute Minima and Maxima, Tangents To Curves

Paper: Part A-Calc / Series: 2006-Form-B / Difficulty: Medium / Question Number: 2



Graph of f'

2. Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.

   (a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

   (b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

   (c) Write an equation for the line tangent to the graph of f at $x = 2$.

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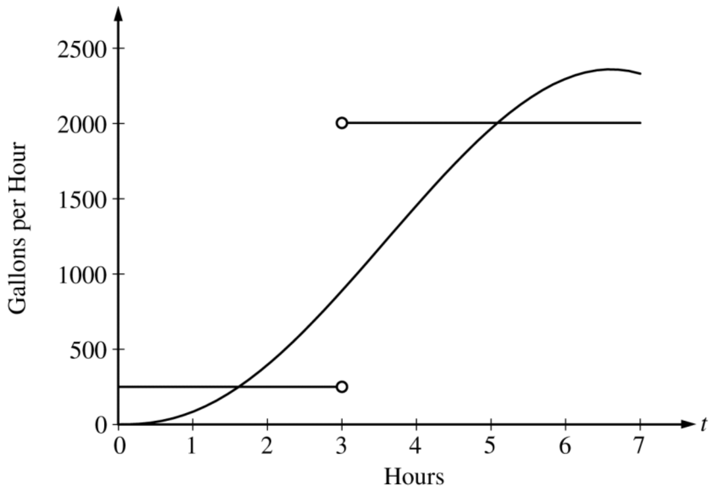
Question 12

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Accumulation of Change, Increasing/Decreasing , Modelling Situations, Derivative Graphs, Global or Absolute Minima and Maxima

Paper: Part A-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 2



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

   (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

   (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

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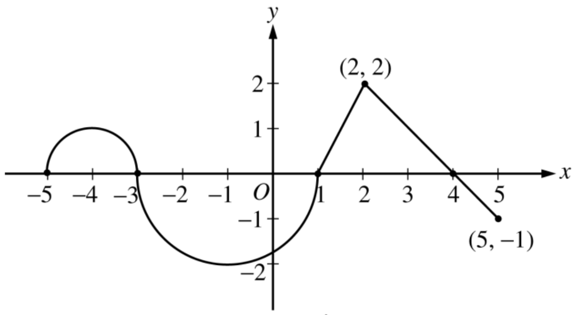
Question 13

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Applications of Integration

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Points Of Inflection, Concavity, Increasing/Decreasing , Derivative Graphs, Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Easy / Question Number: 4



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

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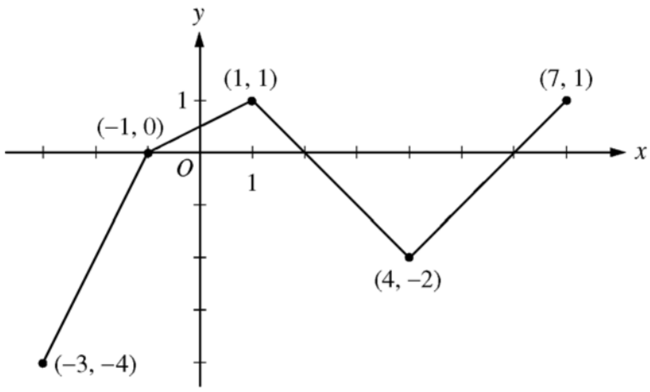
Question 14

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Derivative Graphs, Points Of Inflection, Global or Absolute Minima and Maxima, Rates of Change (Average), Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 5



Graph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

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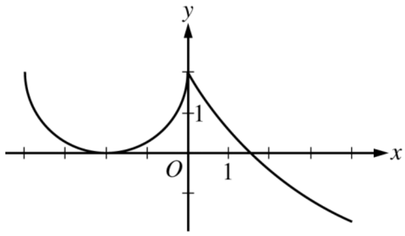
Question 15

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Points Of Inflection, Integration Technique – Geometric Areas, Derivative Graphs, Global or Absolute Minima and Maxima, Differentiation Technique – Exponentials, Integration Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: / Question Number: 6



Graph of f'

6. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}$.

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

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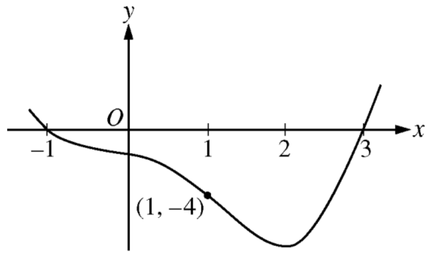
Question 16

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Tangents To Curves, Local or Relative Minima and Maxima, Increasing/Decreasing , Rates of Change (Average), Derivative Graphs

Paper: Part B-Non-Calc / Series: 2009-Form-B / Difficulty: Hard / Question Number: 5



Graph of f'

5. Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

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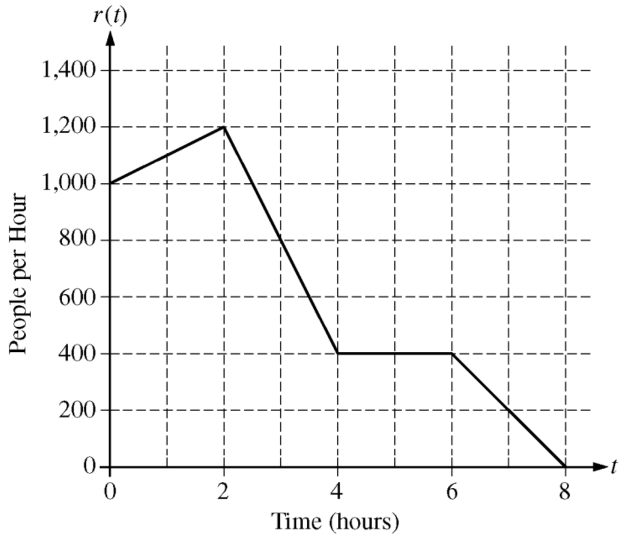
Question 17

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Increasing/Decreasing , Global or Absolute Minima and Maxima, Modelling Situations, Integration Technique – Geometric Areas, Derivative Graphs

Paper: Part A-Calc / Series: 2010 / Difficulty: Easy / Question Number: 3



3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

   (a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

   (b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

   (c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

   (d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

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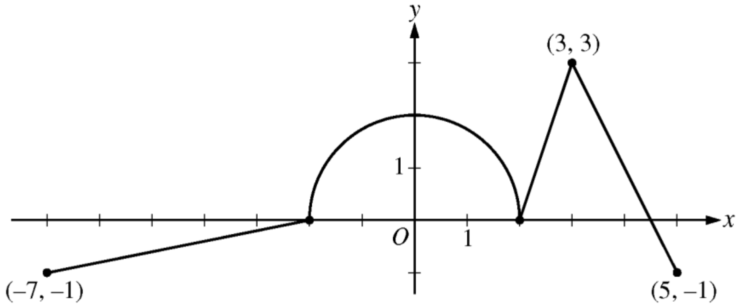
Question 18

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation

Subtopics: Derivative Graphs, Integration Technique – Geometric Areas, Points Of Inflection, Local or Relative Minima and Maxima

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Somewhat Challenging / Question Number: 5



Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

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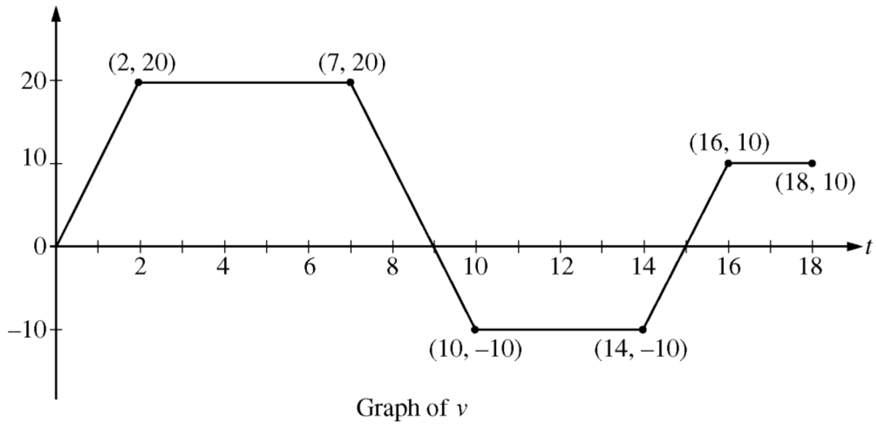
Question 19

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation, Integration

Subtopics: Kinematics (Displacement, Velocity, and Acceleration), Global or Absolute Minima and Maxima, Derivative Graphs, Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (Second)

Paper: Part B-Non-Calc / Series: 2010-Form-B / Difficulty: Medium / Question Number: 4



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at that time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

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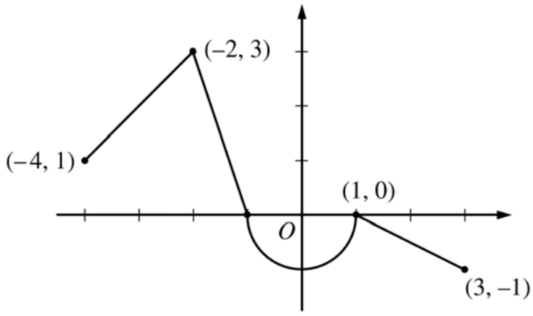
Question 20

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (Second), Local or Relative Minima and Maxima, Points Of Inflection, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Medium / Question Number: 3



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

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Question 21

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differential Equations

Subtopics: Derivative Graphs, Concavity, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Initial Conditions in Differential Equation, Particular Solution of Differential Equation

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Somewhat Challenging / Question Number: 5

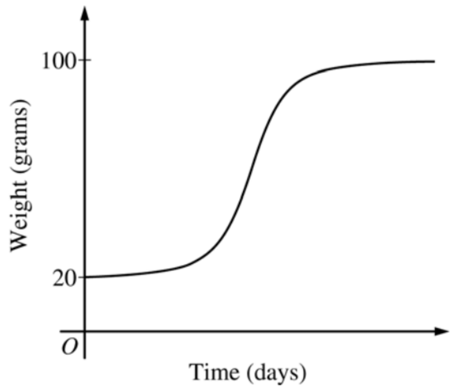
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

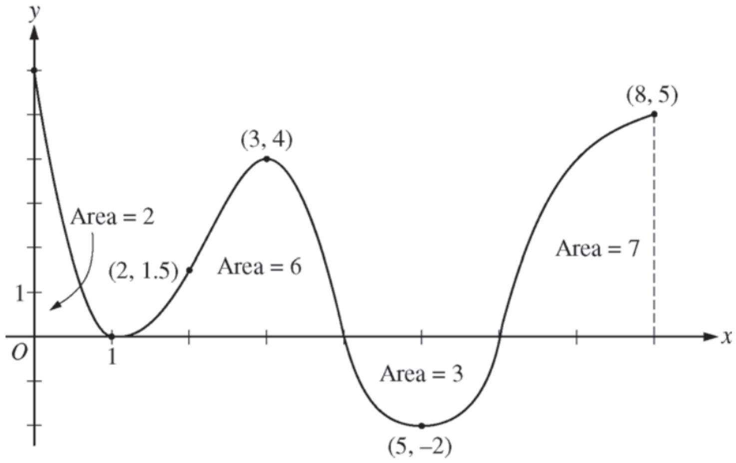
Question 22

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Concavity, Increasing/Decreasing , Implicit Differentiation, Tangents To Curves, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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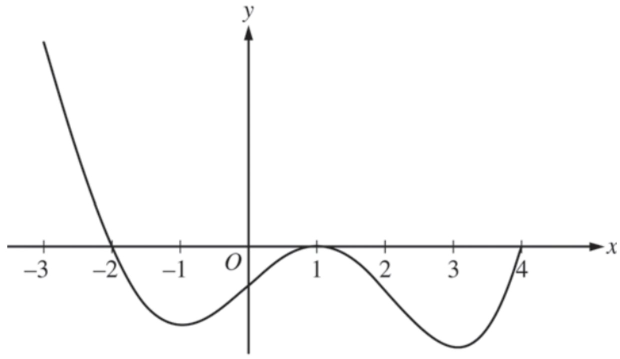
Question 23

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Increasing/Decreasing , Concavity, Points Of Inflection, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Easy / Question Number: 5



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

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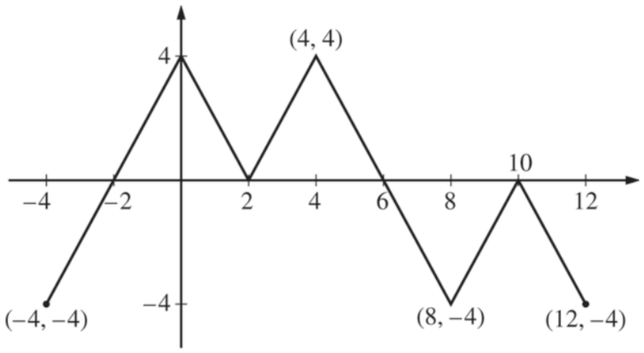
Question 24

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (Second), Points Of Inflection, Global or Absolute Minima and Maxima, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 3



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

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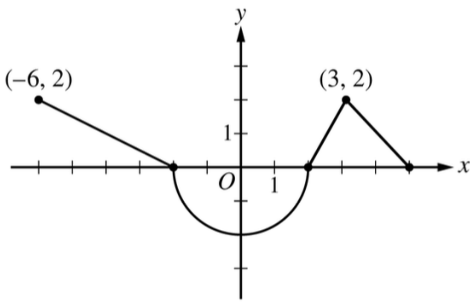
Question 25

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Integration Technique – Geometric Areas, Derivative Graphs, Increasing/Decreasing , Global or Absolute Minima and Maxima, Differentiability

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 3



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

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Question 26

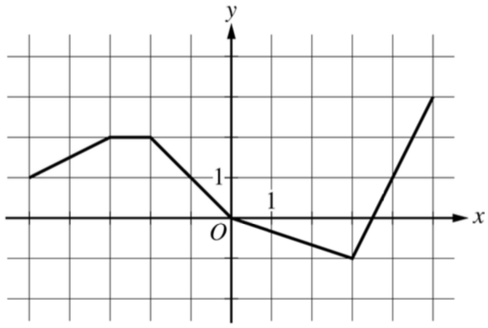
Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Derivative Tables, Tangents To Curves, Differentiation Technique – Chain Rule, Derivative Graphs, Differentiation Technique – Product Rule, Mean Value Theorem, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 6

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

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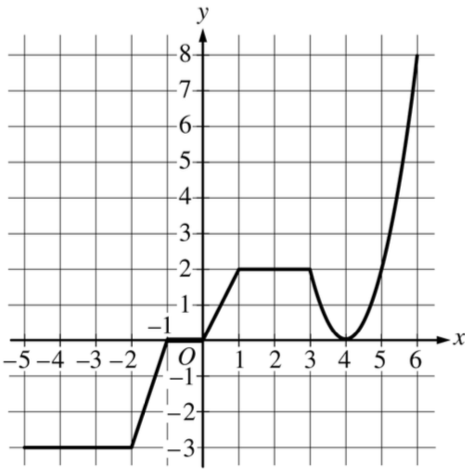
Question 27

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Derivative Graphs, Increasing/Decreasing , Concavity, Points Of Inflection

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Medium / Question Number: 3



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\, dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

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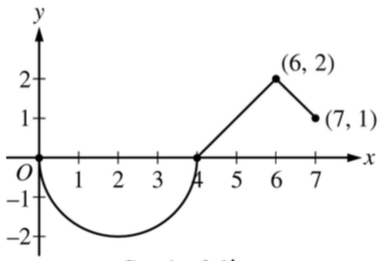
Question 28

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Derivative Graphs, Integration Technique – Geometric Areas, Points Of Inflection, Increasing/Decreasing , Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Easy / Question Number: 3



Graph of f'

3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \le x \le 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.

(a) Find $f(0)$ and $f(5)$.

(b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

(c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \le x \le 7$? Show the analysis that leads to your answer.

(d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \le x \le 7$. Justify your answer.

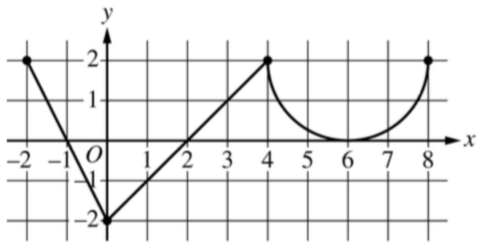
Question 29

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Concavity, Derivative Graphs, Global or Absolute Minima and Maxima, Integration Technique – Geometric Areas, L'Hôpital's Rule, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Medium / Question Number: 4



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

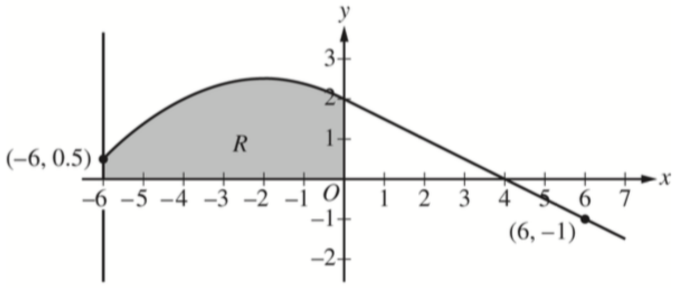
Question 30

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (Second), Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Medium / Question Number: 4



Graph of f

4. The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

(a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

(b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

(c) The function h is defined by $h(x) = \int_{-6}^x f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

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